Mathematics - Course 121

SOLUTIONS TO ASSIGNMENTS

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This Appendix contains 'skeleton' solutions to computational Assignment questions in the text.

## 121.00-3 Probability Problems - Solutions

1. Let M,W denote "man survives", "wife survives", respectively.

(a) $P(M \cap W) = P(M)P(W)$ (PR1)

$= .8 \times .9$

$= .72$

(b) $P(M \cap \overline{W}) = .8 \times .1$ (PR1)

$= .08$

(c) $P(\overline{M} \cap W) = .2 \times .9$ (PR1)

$= .18$

(d) $P(M \cup W) = P(M) + P(W) - P(M)P(W)$ (PR3)

$= .8 + .9 - .8 \times .9$

$= 0.98$

2. (a) Combinations yielding a total of 7 are

(1,6), (6,1) (2,5), (5,2), (3,4), (4,3)

$\therefore$ using geometrical definition of probability,

$$P(\text{sum of } 7) = \frac{6}{36}$$

$$= \frac{1}{6}$$

(b) There are 11 outcomes involving a "1"

$\therefore \quad P(\text{no } 1) = \frac{25}{36}$

(c) Number of outcomes involving exactly one 1 = 10

$\therefore$ P(one 1) $= \frac{10}{36}$

$= \frac{5}{18}$

(d) P(at least one 1) $= \frac{11}{36}$

3. Let 3W denote "3 white balls", etc

(a) No. ways to get 3 of the 5 white balls = ${}_5C_3$

No. ways to get 0 of the 3 black balls = ${}_3C_0$

No. ways to choose 3 balls from the 8 balls = ${}_8C_3$

$$\therefore \quad P(3W) = \frac{{}_5C_3 \times {}_3C_0}{{}_8C_3}$$

$$= \left(\frac{5!}{3!2!} \times \frac{3!}{3!0!}\right) \div \frac{8!}{5!3!}$$

$$= \frac{5}{28}$$

(b) $P(3W \cup 3B) = P(3W) + P(3B)$ (PR4)

$$P(3B) = \frac{{}_5C_0 \times {}_3C_3}{{}_8C_3}$$

$$= \frac{1}{56}$$

$$\therefore P(3W \cup 3B) = \frac{5}{28} + \frac{1}{56}$$

$$= \frac{11}{56}$$

(c) P(at least one white) = 1 - P(OW (PR5)

= 1 - P(3B) (OW $\Longleftrightarrow$ 3B)

$= 1 = \frac{1}{56}$ (from (b))

$= \frac{55}{56}$

4. There are 10 possible last digits for the second number, 9 of which will differ from the last digit of the first number.

$\therefore$ P(different last digits) $= \frac{9}{10}$

5. Let G1 = "first child girl", etc

(a) $P(G2|G1) = P(G2)$ since G1, G2 independent

$$= \frac{1}{2}$$

(b) $$P(\text{2 girls}|\text{at least one girl}) = \frac{P(\text{2 girls} \cap \text{at least one girl})}{P(\text{at least one girl})}$$

$$= \frac{P(\text{2 girls})}{1-P(\text{2 boys})}$$

$$= \frac{\frac{1}{4}}{1 - \frac{1}{4}}$$

$$= \underline{\underline{\frac{1}{3}}}$$

6. (a) $P(6 \cap H \cap KS) = P(6)P(H)P(KS)$ (PR1)

$$= \frac{1}{6} \times \frac{1}{2} \times \frac{1}{52}$$

$$= \underline{\underline{\frac{1}{624}}}$$

(b) $P(\overline{6 \cap H \cap KS}) = 1 - \frac{1}{624}$ (PR5)

$$= \underline{\underline{\frac{623}{624}}}$$

(c) $P(\text{odd} \cap T \cap \text{Club}) = P(\text{odd})P(T)P(\text{Club})$ (PR1)

$$= \frac{3}{6} \times \frac{1}{2} \times \frac{13}{52}$$

$$= \underline{\underline{\frac{1}{16}}}$$

(d) $P[(6 \cup H) \cap Q] = P(6 \cup H)P(Q)$ (PR1)

$= [P(6) + P(H) - P(6)P(H)]P(Q)$ (PR3)

$= (\frac{1}{6} + \frac{1}{2} - \frac{1}{6} \times \frac{1}{2})\frac{4}{52}$

$= \underline{\underline{\frac{7}{156}}}$

7. $P(3R \cap 2B \cap 0W) = \dfrac{{}_7C_3 \times {}_4C_2 \times {}_3C_0}{{}_{14}C_5}$

$= \underline{\underline{\frac{15}{143}}}$

8. (a) Outcomes in $E_1$ are (1,4), (4,1), (2,3), (3,2)

$\therefore$ $P(E_1) = \frac{4}{36}$

$= \underline{\underline{\frac{1}{9}}}$

(b) $P(E_2) = P(R4 \cup G4)$ ($R4 \equiv$ red 4, etc)

$= P(R4) + P(G4) - P(R4)P(G4)$

$= \frac{1}{6} + \frac{1}{6} - \frac{1}{6} \times \frac{1}{6}$

$= \underline{\underline{\frac{11}{36}}}$

(c) $P(E_3) = 0$ (impossible event)

(d) $P(R4 \cap G5) = P(R4)P(G5)$ (PR1)

$= \frac{1}{6} \times \frac{1}{6}$

$= \underline{\underline{\frac{1}{36}}}$

(e) $P(R4 \cup G5) = P(R4) + P(G5) - P(R4)P(G5)$ (PR3)

$$= \frac{1}{6} + \frac{1}{6} - \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{11}{36}$$

9. (a) P(At least one person gets all the cards of one suit)

$$= \frac{4^2 \times {}_{13}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13}}{{}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13}}$$

$$= \underline{\underline{2.52 \times 10^{-11}}}$$

(b) P(one person gets 4 aces, 4 kings, 4 queens)

$$= \frac{4 \times {}_{12}C_{12} \times {}_{40}C_{1} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13}}{{}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13}}$$

$$= \underline{\underline{2.52 \times 10^{-10}}}$$

10. Let E, A represent failure of engine, airframe, respectively. Then

$$P(\text{failure}) = P(E \cup A)$$

$$= P(E) + P(A) - P(E)P(A)$$

$$= 0.002 + 0.0007 - 0.002 \times 0.0007$$

$$= \underline{\underline{0.003}}$$

11. $$P(\text{2 heads} \mid \text{at least 1H}) = \frac{P(\text{2 heads} \cap \text{at least 1H})}{P(\text{at least 1H})}$$

$$= \frac{P(\text{2 heads})}{1-P(\text{2 tails})}$$

$$= \frac{\frac{1}{4}}{1 - \frac{1}{4}}$$

$$= \underline{\underline{\frac{1}{3}}}$$

12. Let A, B, C, S represent failure of component A, B, C, and system, respectively. Then

$P(S) = P(A \cup (B \cap C))$

$= P(A) + P(B \cap C) - P(A)P(B \cap C)$ (PR3)

$= P(A) + P(B)P(C) - P(A)P(B)P(C)$ (PR1)

$= 0.02 + 0.08 \times 0.10 - 0.02 \times 0.08 \times 0.10$

$= \underline{\underline{0.03}}$

13. (a) $N(A) = 31$

(b) $N(B) = 39$

(c) $N(C) = 30$

(d) $N(A \cap B) = 16$

(e) $N(A \cap C) = 12$

(f) $N(A \cap B \cap C) = 4$

(g) $N(A \cup B) = 54$

(h) $N(B \cup C) = 57$

(i) $N(A \cup B \cup C) = 64$

(j) $N(B \cap (A \cup C)) = 24$

14. (a) $P(B) = \frac{39}{75}$

(b) $P(A) = \frac{31}{75}$

(c) $P(B \cap \bar{C}) = \frac{9}{25}$

(d) $P(\bar{B} \cap A \cap C) = \frac{8}{75}$

(e) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$= \frac{16}{31}$

(f) $P(C|B) = \frac{P(B \cap C)}{P(B)}$

$= \frac{12}{39}$

$= \frac{4}{13}$

(g) $P(A \cup C|B) = \frac{P([A \cup C] \cap B)}{P(B)}$

$= \frac{24}{39}$

$= \frac{8}{13}$

(h) $P(B \cap C|\bar{A}) = \frac{P([B \cap C] \cap \bar{A})}{P(\bar{A})}$

$= \frac{8}{75 - 31}$

$= \frac{2}{11}$

(i) $P(\overline{B}|A\cap C) = \frac{P(\overline{B}\cap[A\cap C])}{P(A\cap C)}$

$= \frac{8}{12}$

$= \frac{2}{3}$

15. Let A, B, C represent solution by A, B, C, respectively.

Then P (solution) $= P(A\cup B\cup C)$

$= 1 - P(\overline{A}\cap\overline{B}\cap\overline{C})$ (PR5)

$= 1 - P(\overline{A})P(\overline{B})P(\overline{C})$ (PR1)

$= 1 - \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2}$

$= \frac{3}{4}$

16. <u>One Die</u>

$E(x) = \sum_i x_i P_i$ (x represents gain) (PR9)

$= \$1(\frac{1}{6}) + 2\$(\frac{1}{6}) + 3\$(\frac{1}{6}) + (-\$4)\frac{1}{6} + 5(\frac{1}{6}) + (-\$6)\frac{1}{6}$

$= \$\frac{1}{6}$

<u>Two Dice</u>

$E(x) = \$2(\frac{1}{36}) + (\$3)\frac{2}{36} + (-\$4)\frac{3}{36} + (\$5)\frac{4}{36} + (-\$6)\frac{5}{36} + (\$7)\frac{6}{36}$

$+ (-\$8)\frac{5}{36} + (-\$9)\frac{4}{36} + (-\$10)\frac{3}{36} + (\$11)\frac{2}{36} + (-\$12)\frac{1}{36}$

$= -\$\frac{68}{36}$ (-\$1.89)

$\therefore$ Student should play game with one die, but not with two dice.

17. P(at least one number > 4)

$= 1 - P(\text{both numbers} \leq 4)$ (PR5)

$= 1 - (\frac{4}{6})^2$

$= \underline{\underline{\frac{5}{9}}}$

18. Let A, B denote "target hit by A, B", respectively. Then

(a) $P(A \cap B) = P(A)P(B)$ (PR1)

$= \frac{1}{2}(\frac{1}{4})$

$= \frac{1}{8}$

(b) $P(A \cap \bar{B}) = \frac{1}{2}(\frac{3}{4})$

$= \frac{3}{8}$

(c) $P(\bar{A} \cap B) = \frac{1}{2}(\frac{1}{4})$

$= \frac{1}{8}$

(d) $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

$= \frac{1}{2}(\frac{3}{4})$

$= \frac{3}{8}$

(e) Suppose B must fire n times.

Then P(target missed altogether) $\leq 0.1$

$\Rightarrow \quad P(\bar{A} \cap \bar{B}_1 \cap \bar{B}_2 \cap \ldots \cap \bar{B}_n) \leq 0.1$

ie, $P(\bar{A})[P(B)]^n \leq 0.1$ (PR1)

ie, $\frac{1}{2}(\frac{3}{4})^n \leq 0.1$

ie, $n \log 0.75 \leqslant \log 0.2$

ie, $n \geqslant \frac{\log 0.2}{\log 0.75}$

ie, $n \geq 5.6$

ie, B must fire 6 times before probability that target is hit exceeds 90%.

19. P(both numbers odd)

$$= \frac{\text{Number odd-odd combinations}}{\text{Number odd-odd + Number even-even}}$$

$$= \frac{5^2}{5^2 + 4^2}$$

$$= \frac{25}{41}$$

20. $E(x) = \sum_i x_i P_i$ (x represents gain)

$= (\$1)\ P(1\ \text{head}) + (\$2)P(2\ \text{heads}) + (-\$5)P(2\ \text{tails})$

$= (\$1)(\frac{1}{2}) + (\$2)\frac{1}{4} + (-\$5)(\frac{1}{4})$

$= \underline{\underline{-\$\frac{1}{4}}}$

$\therefore$ he should not be playing the game.

21. Let A, B represent item manufactured by machine #1, 2, respectively, and D represent item defective. Then

$$P(D) = P(D|A)P(A) + P(D|B)P(B) \qquad \text{(PR8)}$$

$$= 0.05 \times 0.70 + 0.08 \times 0.30$$

$$= \underline{\underline{0.059}}$$

22. (a) $$P(A \cap B) = P(A|B)P(B) \qquad \text{(PR7)}$$

$$= \frac{6}{11}\left(\frac{11}{36}\right)$$

$$= \underline{\underline{\frac{1}{6}}}$$

(b) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{11}{36} - \frac{1}{6}$$

$$= \underline{\underline{\frac{23}{36}}}$$

(c) $$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= \frac{1}{2} - \frac{1}{6}$$

$$= \underline{\underline{\frac{1}{3}}}$$

(d) $$P(B \cap \bar{A}) = P(B) - P(B \cap A)$$

$$= \frac{11}{36} - \frac{1}{6}$$

$$= \underline{\underline{\frac{5}{36}}}$$

## 121.00-5 Safety Systems Analysis - Solutions to Sample Problems

1. $Q = \lambda\frac{T}{2}$

$= \frac{5}{12 \times 30} \times \frac{\frac{1}{2}}{2}$

$= 3 \times 10^{-3}$

2. $Q = \lambda\frac{T}{2}$

$= \frac{3}{6 \times 12} \times \frac{\frac{1}{2} \times \frac{1}{52}}{2}$

$= 2 \times 10^{-4}$

3. $Q_s = Q_1 + Q_2 - Q_1Q_2$

$= 1.7 \times 10^{-2}$

4. $Q_s = Q_p^{\,2}$

$= 4 \times 10^{-4}$

5. $Q = \lambda\frac{T}{2}$

$= \frac{50}{15 \times 10} \times \frac{\frac{1}{52}}{2}$

$= 3 \times 10^{-3}$

6. $T = \frac{2Q}{\lambda}$

$= \frac{2 \times 1.0 \times 10^{-2}}{\frac{15}{5 \times 12}}$

$= 0.08$ years or 4.2 weeks.

ie, the system should be tested every 4 weeks.

7. $$Q = \lambda\frac{T}{2}$$

$$= \frac{10}{12 \times 8} \times \frac{\frac{1}{12}}{2}$$

$$= \underline{\underline{4 \times 10^{-3}}}$$

8. $$AR = \lambda_R\lambda_P\frac{T_P}{2}$$

$$= \frac{3}{9} \times \frac{50}{9} \times \frac{\frac{1}{3} \times \frac{1}{365}}{2}$$

$$= \underline{\underline{8 \times 10^{-4}}}$$

9. (a) $AR = \lambda_R(Q_p + Q_{CT} - 2Q_PQ_{CT})$ (exclusive "or")

$$= 0.3(2 \times 10^{-3} + 5 \times 10^{-3} -2 \times 2 \times 10^{-3} \times 5 \times 10^{-3})$$

$$= \underline{\underline{2 \times 10^{-3}}}$$

(b) $$AR = \lambda_RQ_PQ_{CT}$$

$$= 0.3 \times 2 \times 10^{-3} \times 5 \times 10^{-3}$$

$$= \underline{\underline{3 \times 10^{-6}}}$$

10. (a) $$Q = \lambda\frac{T}{2}$$

$$= \frac{8}{6 \times 15} \times \frac{\frac{1}{12}}{2}$$

$$= \underline{\underline{4 \times 10^{-3}}}$$

(b) Test daily.

(c) $T = \frac{2Q}{\lambda}$

$$= \frac{2 \times 10^{-2}}{\frac{8}{6 \times 15}}$$

$= 0.225y$ or $\underline{\underline{12 \text{ weeks}}}$

11. $Q_S = P(\bar{A} \cup [\overline{BC}D \cup \bar{B}C\bar{D} \cup B\overline{CD} \cup \overline{BCD}]$

$= Q_A + [3Q_B^2 R_B + Q_B^3] - Q_A\ [\quad]$

$= 0.05 + [3(.1)^2(.9) + (.1)^3] - 0.05 \times [\quad]$

$= \underline{\underline{0.08}}$

12. (a) $Q$ = fraction of time pump unavailable

$$= \frac{124 \text{ h}}{5 \times 365 \times 24 \text{ h}}$$

$= \underline{\underline{2.8 \times 10^{-3}}}$

(b) $Q_S = P[(\text{exactly 2 pumps fail}) \cup (\text{exactly 3 pumps fail})]$

$= 3C_2Q^2R + {}_3C_3Q^3)$

$= 3(.0028)^2(1-.0028) + (.0028)^3$

$= \underline{\underline{2.4 \times 10^{-5}}}$

13. (a) $P_{LS} = \lambda\frac{T}{2} = .02 \times \frac{\frac{1}{12}}{2} = \frac{.02}{24} = \frac{.01}{12}$

$P_{PS} = \lambda\frac{T}{2} = .02 \times \frac{\frac{1}{12}}{2} = \frac{.01}{12}$

$P_{PV} = \lambda\frac{T}{2} = .05 \times \frac{\frac{1}{12}}{2} = \frac{.05}{24}$

$Q_S = (\frac{.01}{12})^2 + (\frac{.01}{12}) + 5(\frac{.05}{24})$

$= 0.01$

(b) $P_{LS} \longrightarrow .04 \times \frac{\frac{1}{12}}{2} = \frac{.01}{6}$

$\therefore \; Q_S = (\frac{.01}{6})^2 + (\frac{.01}{12}) + 5(\frac{.05}{24})$

$= 0.01$ (ie, virtually no change since LS contributes negligibly to $Q_S$ in both cases.)

14. Fraction of time system unavailable,

$Q = \lambda\frac{T}{2}$

$= \frac{20}{4} \times \frac{\frac{1}{365}}{2}$

$= 0.007$

$< 1\%$

$\therefore$ probability of fault existing at any given instant $< 1\%$.

$\because$ $Q \propto T$ and $T = 1$ week is seven times greater than $T = 1$ day,

$\therefore$ unavailability would be seven times greater with same $\lambda$ and weekly testing.

15. (a)

$$AR = \lambda_R Q_P$$

$$= \lambda_R \lambda_P \frac{T_P}{2}$$

$$= \frac{2}{6} \times \frac{3}{6} \times \frac{\frac{1}{365}}{2}$$

$$= \underline{\underline{2.3 \times 10^{-4}}}$$

(b) Unavailability of containment,

$$Q_C = Q_1 + Q_2 - Q_1Q_2,$$

where $Q_1 \equiv$ unavailability of air locks

$$= \frac{40 \text{ hr}}{6 \times 365 \times 24 \text{ hr}}$$

$$= 7.6 \times 10^{-4}$$

and $Q_2 \equiv$ unavailability of logic system

$$= \lambda\frac{T}{2}$$

$$= \frac{4}{6} \times \frac{\frac{1}{52}}{2}$$

$$= 6.4 \times 10^{-3}$$

$\therefore Q_2 \equiv$ unavailability of logic system

$$= \lambda\frac{T}{2}$$

$$= \frac{4}{6} \times \frac{\frac{1}{52}}{2}$$

$$= 6.4 \times 10^{-3}$$

$$\therefore Q_C = 7.6 \times 10^{-4} + 6.4 \times 10^{-3} = 7.2 \times 10^{-3}$$

$$\therefore \quad P(\text{runaway + release}) = AR \times Q_C$$

$$= 2.3 \times 10^{-4} \times 7.2 \times 10^{-3}$$

$$= \underline{\underline{1.7 \times 10^{-6}}}$$

16. (a) The reliability of safety systems can be increased by:

    (i) use of redundant components.

    (ii) preventive replacement of components prior to wearout.

    (iii) testing more frequently.

17. Reactor safety systems should be tested routinely.

    (a) to detect and repair/replace faulty components.

    (b) to <u>maintain</u> system reliability.

    ($R = 1 - Q = 1 - \lambda\frac{T}{2}$. $\therefore$ the shorter T, the greater R)

    (c) to demonstrate whether or not reliability is meeting target, so that corrective action (eg, upgrading system, or more frequent testing) can be taken if it is not.

    (d) to satisfy AECB license requirements.

18. (a) Expected runaway frequency,

$$\lambda_{rw} = \lambda_R \lambda_P \frac{T_P}{2}$$

$$= \frac{3}{5} \times \frac{2}{5} \times \frac{\frac{1}{365}}{2}$$

$$= \underline{\underline{3 \times 10^{-4}}}$$

(b) Probability of one or more LOR's/y,

$$Q(1) = 1 - R(1)$$

$$= 1 - e^{-\lambda_R \times 1}$$

$$= 1 - e^{-0.6}$$

$$= \underline{\underline{0.45}}$$

19. Let $Q_v$, $Q_l$, $Q_s$ represent unreliabilities of a value, a line, and the system, respectively.

(a) (i) $Q_v = \lambda\frac{T}{2}$

$$= \frac{6}{6 \times 5} \times \frac{\frac{1}{2} \times \frac{1}{52}}{2}$$

$$= 1.0 \times 10^{-3} \qquad (9.6 \times 10^{-4})$$

(ii) $Q_l$ = prob. either upper or lower valve fails

$$= Q_v + Q_v - Q_v^2$$

$$= 2(9.6 \times 10^{-4})$$

$$= \underline{\underline{2 \times 10^{-3}}} \qquad (1.9 \times 10^{-3})$$

(b) $Q_s$ = prob. all three lines fail

$$= Q_\ell^3$$

$$= (1.9 \times 10^{-3})^3$$

$$= \underline{\underline{7 \times 10^{-9}}}$$

20. (a) Unreliability of a dump channel,

$$Q_C = \lambda\frac{T}{2}$$

$$= \frac{4}{5 \times 3} \times \frac{\frac{1}{3} \times \frac{1}{52}}{2}$$

$$= \underline{\underline{9 \times 10^{-4}}} \qquad (8.55 \times 10^{-4})$$

(b) $Q_S$ = prob. of 2 or 3 channels failing at once,

$$= {}_3C_2 Q_C^2(1 - Q_C) + {}_3C_3 Q_C^3$$

$$= 3(8.5 \times 10^{-4})^2$$

$$= \underline{\underline{2 \times 10^{-6}}} \qquad (2.2 \times 10^{-6})$$

(c) F valves <u>should</u> be left open.

<u>Assuming F valves open:</u>

$Q_S$ = prob. both D, E fail

$$= Q_C^2$$

$$= (8.5 \times 10^{-4})^2$$

$$= \underline{\underline{7 \times 10^{-7}}}$$

<u>Assuming F valves shut:</u>

$Q_S$ = prob. either D or E fails or both

$$= Q_C + Q_C - Q_C{}^2$$

$$= 2(8.5 \times 10^{-4})$$

$$= \underline{\underline{1.7 \times 10^{-3}}}$$

21. Let $Q_C$, $Q_V$ represent unreliability of control channel, mechanics of a valve, respectively.

(a) (i) $Q_C = \lambda\frac{T}{2}$

$$= \frac{4}{5 \times 3} \quad \frac{\frac{1}{3} \times \frac{1}{52}}{2}$$

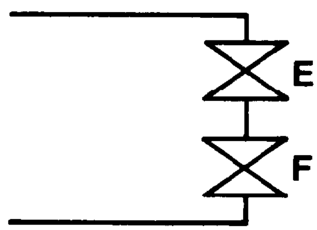
$$= \underline{\underline{8.5 \times 10^{-4}}}$$

(ii) $Q_V = \lambda\frac{T}{2}$

$$= \frac{7}{5 \times 6} \times \frac{\frac{1}{2} \times \frac{1}{52}}{2}$$

$$= \underline{\underline{1.1 \times 10^{-3}}}$$

(b) (i) Suppose channel D failed. Then system effectively as shown:



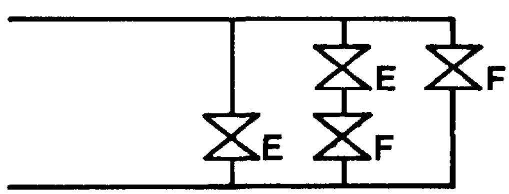

$Q_S$ = prob. either E or F fails or either valve fails mechanically.

$$= (2Q_C - Q_C{}^2) + (2Q_V - Q_V{}^2) - (2Q_C - Q_C{}^2)(2Q_V - Q_V{}^2)$$

$$= 2(Q_C + Q_V)$$

$$= \underline{\underline{4 \times 10^{-3}}} \qquad (3.9 \times 10^{-3})$$

(ii) If D valves are opened then system is effectively as shown:




Failure modes: 2 channels (1 way)

1 channel, 1 valve (2 ways)

3 valves (2 ways)

+ higher order modes

$Q_S \doteq$ prob. of failing 2 channels + prob. of failing 1 channel and 1 valve

$$\doteq Q_C^{\,2} + 2\,Q_C Q_V$$

$$= (8.5 \times 10^{-4})^2 + 2(8.5 \times 10^{-4})(1.1 \times 10^{-3})$$

$$= \underline{\underline{3 \times 10^{-6}}}$$

## 121.00-6 The Binominal Distribution and Power System Reliability

1. $Q_S = {}_{14}C_3 q^3 r^{11} + {}_{14}C_4 q^4 r^{10} + {}_{14}C_5 q^5 r^9 + \ldots + {}_{14}C_{14} q^{14}$

$$= \frac{14!}{11!3!}(.01)^3(.99)^{11} + \frac{14!}{10!4!}(.01)^4(.99)^{10} + \ldots + (.01)^{14}$$

$$= 3.26 \times 10^{-4} + 9.1 \times 10^{-6} + \ldots + 1 \times 10^{-28}$$

$$= 3.4 \times 10^{-4}$$

Note system unavailability is greater than that in Example 5. Even though there are more valves in this system, the redundancy is decreased, because at least 12 of 14 valves are required for this system's success as compared with at least 6 of 8 valves in the system of Example 5.

2. (a) <u>Capacity Outage Probability Distribution Table - Generators A, B</u>

| k | Outage $O_k$ Units | Outage $O_k$ Capacity | Probability $P_k$ | Fraction of time $O_k$ causes load loss $t_k$ (y/y) | $P_k t_k$ |
|---|---|---|---|---|---|
| 1 | none | 0 | 0.912 | 0 | 0 |
| 2 | A | 50 | 0.048 | 0.133 | 0.0064 |
| 3 | B | 60 | 0.038 | 0.400 | 0.0152 |
| 4 | A,B | 110 | 0.002 | 1 | 0.0020 |

$$ELC = \sum_{k=1}^{4} P_k t_k$$

$$= 0.0236$$

ie, there is a load curtailment about 2.4% of the time

2. (b) Capacity Outage Probability Distribution Table

| k | Outage $O_k$ | | Probability $P_k$ | Fraction of time $O_k$ causes load loss $t_k$ (y/y) | $P_k t_k$ |
|---|---|---|---|---|---|
| | Units | Capacity | | | |
| 1 | none | 0 | 0.87552 | 0 | 0 |
| 2 | A | 50 | 0.04608 | 0 | 0 |
| 3 | B | 60 | 0.03648 | 0.13333 | 0.00486 |
| 4 | C | 10 | 0.03648 | 0 | 0 |
| 5 | A,B | 110 | 0.00192 | 1 | 0.00192 |
| 6 | A,C | 60 | 0.00192 | 0.13333 | 0.00026 |
| 7 | B,C | 70 | 0.00152 | 0.40000 | 0.00061 |
| 8 | A,B,C | 120 | 0.00008 | 1 | 0.00008 |

$$ELC = \sum_{k=1}^{8} P_k t_k$$

ie, there is a load curtailment now only 0.77% of the time. Thus an increase of about 9% in generating capacity has reduced the ELC by a factor of about 3. This significant improvement occurs because the system can now tolerate a failure of generator A without load loss.

3. Forced Outage Rate = 0.015

(a) 3 x 100% transformer.

| No OUT | Cap OUT | Probability | EXP% Load Curtailment |
|---|---|---|---|
| 0 | 0 | $0.985^3 = 0.95567$ | 0 |
| 1 | 0 | $3 \times .985^2 \times .015 = 0.04366$ | 0 |
| 2 | 0 | $3 \times .985 \times .015^2 = 6.649 \times 10^{-4}$ | 0 |
| 3 | 100% | $.015^3 = 3.38 \times 10^{-6}$ | 0.00338 |
| | | | 0.000338 |

Expected hours curtailment = $3.38 \times 10^{-6} \times 8760$ = 0.0296 h

(b) 3 x 90% transformers.

| No OUT | Cap OUT | Probability | Exp% Load Curtailment |
|---|---|---|---|
| 0 | 0 | 0.95567 | 0 |
| 1 | 0 | 0.04366 | 0 |
| 2 | 10 | $6.649 \times 10^{-4}$ | 0.006649 |
| 3 | 100 | $3.38 \times 10^{-6}$ | 0.000338 |
| | | | 0.006987 |

Prob. (2 out) + Prob. (3 out) = 0.000668

Expected hour curtailment = 0.000668 x 8760

= 5.85 h

(c) 3 x 50% transformers.

| No OUT | Cap OUT | Probability | Exp% Load Curtailment |
|---|---|---|---|
| 0 | 0 | .95567 | 0 |
| 1 | 0 | .04366 | 0 |
| 2 | 50 | $6.649 \times 10^{-4}$ | 0.033244 |
| 3 | 100 | $3.38 \times 10^{-6}$ | 0.000338 |
| | | | 0.033582 |

Prob. (2 out) + Prob (3 out) = 0.000668

$\therefore$ Expected hour curtailment = 5.85 h

(d) 4 x 33-1/3% transformers

| No OUT | Cap OUT | Probability | Exp% Load Curtailment |
|---|---|---|---|
| 0 | 0 | $.985^4$ | |
| 1 | 0 | $4 \times .98^3 \times .015$ | |
| 2 | 33-1/3 | $6 \times .985^2 \times .015^2 = 1.3098 \times 10^{-3}$ | 0.04366 |
| 3 | 66-2/3 | $4 \times .985 \times .015^3 = 1.3297 \times 10^{-5}$ | 0.000887 |
| 4 | 100 | $.015^4 = 5.0625 \times 10^{-8}$ | 0.000005 |
| | | | 0.044552 |

Prob. (2 out) + Prob. (3 out) + Prob. (4 out) = 0.001323

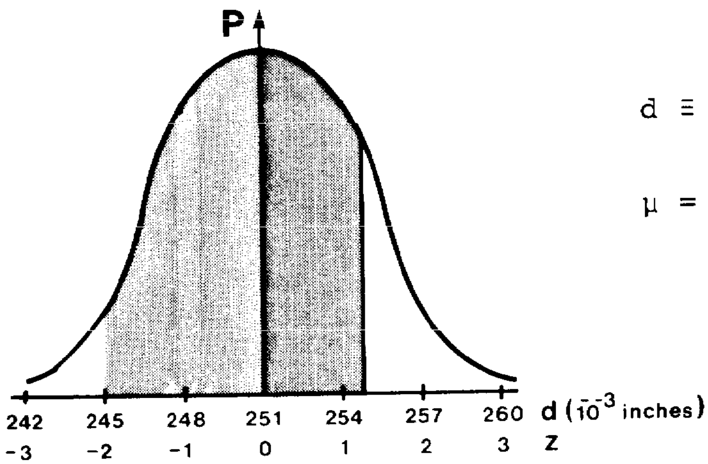
$\therefore$ Exp hour curtailment = 0.001323 x 8760 = 11.6 h

Summary of Results

| System | Exp% Load Curtailment | Exp Load Curtailment h/y |
|---|---|---|
| 3 x 100% | 0.000338 | 0.03 |
| 3 x 90% | 0.00699 | 5.9 |
| 3 x 50% | 0.034 | 5.9 |
| 3 x 33-1/3% | 0.045 | 11.6 |

## 121.00-7 The Normal Distribution and Applications

1.




d ≡ inner diameter of washers in 0.001 inches

$\mu = 251;\ \sigma = 3$

$$P(245 < d < 255) = P\left(\frac{245 - 251}{3} < z < \frac{255 - 251}{3}\right)$$

$$= P(-2 < z < 1.33)$$

$$= F(1.33) - F(-2)$$

$$= F(1.33) - [1 - F(2)]$$

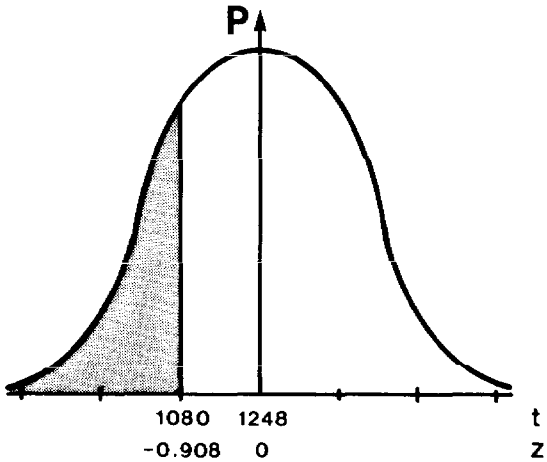
$$= 0.9082 - [1 - 0.9772]^*$$

$$= 0.8854$$

∴ 88.5% of the washers will be within specifications

*From the Normal Distribution Table.

2.




t ≡ battery lifetime in days

$\mu = 1248$

$\sigma = 185$

36 months = 1080 days

$$P(t \leq 1080) = P\left(z \leq \frac{1080 - 1248}{185}\right)$$

$$= P(z \leq -0.908)$$

$$= F(-0.908)$$

$$= 1 - F(0.908)$$

$$= 1 - 0.819$$

$$= 0.181$$

∴ 18.1% of the batteries will have to be replaced.

## 121.00-8 Basic Reliability Concepts

1. (a) $R_S = R_A R_B R_C$

$$= e^{-\alpha t} e^{-\beta t} e^{-\gamma t}$$

$$= e^{-(\alpha + \beta + \gamma)t}$$

(b) $R_S = 1 - Q_A Q_B Q_C$

$$= 1 - (1 - e^{-\alpha t})(1 - e^{-\beta t})(1 - e^{-\gamma t})$$

(c) $R_S = R_C(R_A + R_B - R_A R_B)$

$$= e^{-\gamma t}(e^{-\alpha t} + e^{-\beta t} - e^{-(\alpha + \beta)t})$$

$$= e^{-(\alpha + \beta + \gamma)t} \; (e^{\beta t} + e^{\alpha t} - 1)$$

(d) $R_S = R_A R_B + R_C - R_A R_B R_C$

$$= e^{-(\alpha + \beta + \gamma)t} \; (e^{\gamma t} + e^{(\alpha + \beta)t} - 1)$$

2. $R(50) = 0.9 \implies e^{-50\lambda} = 0.9$

$$\therefore R(100) = e^{-100\lambda}$$

$$= (e^{-50\lambda})^2$$

$$= 0.81$$

3. 4 Components in Parallel:

$$R_S = {}_4C_3 p^3 q + {}_4C_4 p^4$$

$$= 4(0.9)^3(0.1) + 1(0.9)^4$$

$$= \underline{0.9477}$$

5 Components:

$$R_S = {}_5C_3 p^3 q^2 + {}_5C_4 p^4 q + {}_5C_5 p^5$$

$$= 10(0.9)^3(0.1)^2 + 5(0.9)^4(0.1) + (0.9)^5$$

$= \underline{0.99144}$ (Reliability improves since system can now tolerate 2 failures)

4. $$R_{S1} = R_{S1}^{B} R_B + R_{S1}^{\overline{B}} Q_B$$

$$= (R_{A'} + R_{B'} - R_{A'}R_{B'})R_B + (R_A R_{A'} + R_C R_{B'} - R_A R_{A'} R_C R_{B'})Q_B$$

$$= (0.9 + 0.9 - 0.81)(0.9) + (0.81 + 0.81) - (0.9)^4)(0.1)$$

$$= 0.98739$$

$$R_{S2} = 1 - Q_{S2}$$

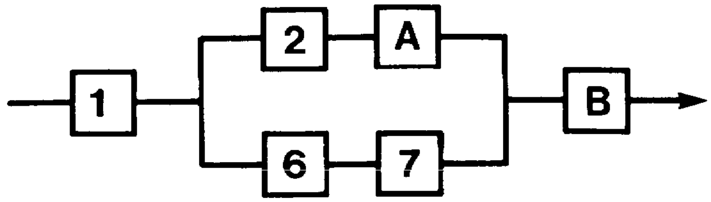
$$= 1 - (Q_A Q_B Q_C + Q_{B'} Q_{A'} - Q_A Q_B Q_C Q_{B'} Q_{A'}$$

$$= 1 - (0.001 + 0.01 - 0.00001)$$

$$= 0.98901$$

System 2 has higher reliability because there are more possible paths, ie, A ⟶ B' and C ⟶ A', which are not open in system 1.

5. Given system $\equiv$




$\equiv$


ie, $R_S = R_1R_CR_B$

where $R_C = (R_2R_A + R_6R_7 - R_2R_AR_6R_7)$,

$R_A = 1 - Q_A = 1 - Q_3Q_4Q_5$, and

$R_B = {}_3C_2R_8^2Q_8 + {}_3C_3R_8^3$

6. Let $E \equiv$ electrical power fails

$P \equiv$ pumps fail

$V \equiv$ valves fail

Then $Q_{SW} = P(E \cup P \cup V)$

$\doteq P(E) + P(P) + P(V)$

$= Q_E + ({}_4C_3Q_P^3R_P + {}_4C_4Q_P^4) + Q_{CV}(Q_L + Q_{BV} - Q_LQ_{BV})$

$\doteq Q_E + 4Q_P^3 + Q_{CV}(Q_L + Q_{BV})$

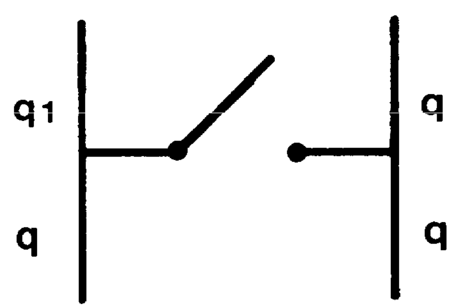
$= \underline{\underline{8 \times 10^{-6}}}$

7. (a) $Q_{S1} = q + q - q^2$

$= \underline{\underline{2 \times 10^{-2}}}$

(b) $Q_{S2} = Q_{S1}^2$

$= \underline{\underline{4 \times 10^{-4}}}$



(c) $$Q_{S3} = Q^{1}_{S3}\, r_1 + Q^{\top}_{S3} q_1 \qquad \text{(Baye's Theorem)}$$

$$= q^2 r + (q + q^2 - q^3)q$$

$$= \underline{\underline{2 \times 10^{-4}}}$$

8. $$Q_{SW} = P(\text{SW fails}|\text{class 4 good})\ R_4$$

$$+ P(\text{SW fails}|\text{class 4 failed})Q_4 \qquad \text{(Baye)}$$

$$= (1 - R_{pumps}\ R_V)R_4 + (1 - R_G R_B R_P^2 R_V)Q_4$$

$$\doteq (Q_{pumps} + Q_V)R_4 + (Q_G + Q_B + 2_{QP} + Q_V)Q_4$$

Now $Q_{pumps} = P(3 \text{ or } 4 \text{ of } 4 \text{ pumps fail})$

$$= {}_4C_3 Q_P^3 R_P + {}_4C_4 Q_P^4$$

$$= 4Q_P^3$$

$$\therefore\ Q_{SW} \doteq 4Q_P^3 + Q_V + (Q_G + Q_B + 2Q_P)Q4$$

$$= 4(6 \times 10^{-4})^3 + 7 \times 10^{-8}$$

$$+ (7 \times 10^{-3} + 2 \times 10^{-2} + 2 \times 6 \times 10^{-4})8 \times 10^{-6}$$

$$= \underline{\underline{3 \times 10^{-7}}}$$

9.

If $S$ ≡ system survives

1FO ≡ switch 1 fails open

2FS ≡ switch 2 fails short

1G ≡ switch 1 is good, etc

then, by Baye's Theorem,

$$P(S) = P(S|1G)P(1G) + P(S|1FO)P(1FO) + P(S|1FS)P(1FS)$$

$$= [1 - P(2FO)]P(1G) + 0 + P(2G)P(1FS)$$

$$= (1 - \frac{\lambda_O}{\lambda} q)r + r(\frac{\lambda_S}{\lambda} q)$$

$$= r + qr\ (\frac{\lambda_S - \lambda_O}{\lambda})$$

$$= r\ (1 + \frac{\lambda_S - \lambda_O}{\lambda}) - r^2\ (\frac{\lambda_S - \lambda_O}{\lambda})$$

ie, $R_S = \frac{2\lambda_S}{\lambda} r + \frac{\lambda_O - \lambda_S}{\lambda} r^2$

The reliability of a single switch system is r.

$$\lambda_O = \lambda_S \Longrightarrow R_S = r$$

Thus when open and short failure modes are equally probable, adding a second switch has no effect on system reliability

$$\lambda_O = 0 \Longrightarrow R_S = 2r - r^2$$

Thus, when the switches can only fail short, system reliability is improved by adding the second switch. Note that the second switch is <u>physically</u> connected in <u>series</u>, but the reliability expression is appropriate for two components in parallel ie, the reliability block diagram would show the two switches in <u>parallel</u>.

$$\lambda_S = 0 \Longrightarrow R_S = r^2$$

Thus, when the switches can only fail open, system reliability is reduced by adding the second switch.

10. (a) $R_S = R_S^D R_D + R_S^{\bar{D}} Q_D$

$= R_D(R_S^{DC} R_C + R_S^{D\bar{C}} Q_C) + Q_D(R_S^{\bar{D}C} R_C + R_S^{\overline{DC}} Q_C)$

$= R_D R_C (R_A R_B R_G + R_F - R_A R_B R_G R_F)$

$+ R_D Q_C (R_A R_B R_G + R_E R_F - R_A R_B R_G R_E R_F)$

$+ Q_D R_C (R_A (R_B R_G + R_F - R_B R_G R_F))$

$+ Q_D Q_C R_A R_B R_G$

$= .81\ (.729 + .9 - 0.6561) + .09\ (.729 + .81 - .59049)$

$+ .09(.9(.81 + .9 - .729)) + .01 \times .729$

$= 0.788049 + .0853659 + 0.079461 + 0.00729$

$= \underline{\underline{0.9601659}}$

(b) $R_S = R_S^A R_A + R_S^{\bar{A}} Q_A$

$= R_A(R_S^{AD} R_D + R_S^{A\bar{D}} Q_D) + Q_A(R_S^{\bar{A}D} R_D + R_S^{\overline{AD}} Q_D)$

$= R_A R_D (R_B R_G + (R_C + R_E - R_C R_E) R_F - R_B R_G (R_C + R_E - R_C R_E) R_F)$

$+ R_A Q_D (R_B R_G + R_C R_F - R_B R_G R_C R_F)$

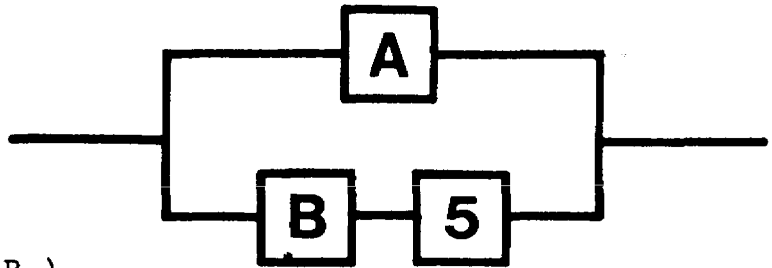
$+ R_D Q_A (R_C + R_E - R_C R_E) R_F$

$+ Q_A Q_D (O)$

$= 0.7932249 + 0.086751 + 0.08019$

$= \underline{\underline{0.9601659}}$

11. (a) Equivalent system:




Then $Q_S = Q_a(1 - R_bR_5)$

$$= (1 - R_1R_2)Q_3[1-({}_4C_2R_4^2Q_4^2 + {}_4C_3R_4^3Q_4 + {}_4C_4R_4^4)R_5]$$

$$= (1 - R_1R_2)Q_3[1 - (6R_4^2Q_4^2 + 4R_4^3Q_4 + R_4^4)R_5]$$

<u>OR</u> $Q_S = (Q_1 + Q_2 - Q_1Q_2)Q_3[4Q_4^3R_4 + Q_4^4 + Q_5$

$\quad - (4Q_4^3R_4 + Q_4^4)Q_5]$

(b) $\underline{\underline{Q_S = 0.00196327}}$

12. (a) Let $Q_{sys}^{1}$, $Q_{sys}^{1fs}$, $Q_{sys}^{1fo}$ represents system unreliability given that diode 1 is up (good), failed short, failed open, respectively. Then by Baye's Theorem,

$$Q_{sys} = Q_{sys}^{1}R_1 + Q_{sys}^{1fs}Q_s + Q_{sys}^{1fo}Q_o$$

$$= Q_sR_1 + 1Q_s + Q\ Q_o$$

$$= (0.01)(0.97) + 0.01 + (0.03)(0.02)$$

$$= 0.0097 + 0.01 + .0006$$

$$= 0.0203$$

(b) $\text{UIR} = \dfrac{0.03}{0.0203}$

$$= 1.48$$

(c) $Q_s = 0.02,\ Q_o = 0.01 \Longrightarrow Q_{sys} = 0.02 \times 0.97 + 0.02 + 0.03 \times 0.01$

$$= 0.0397$$

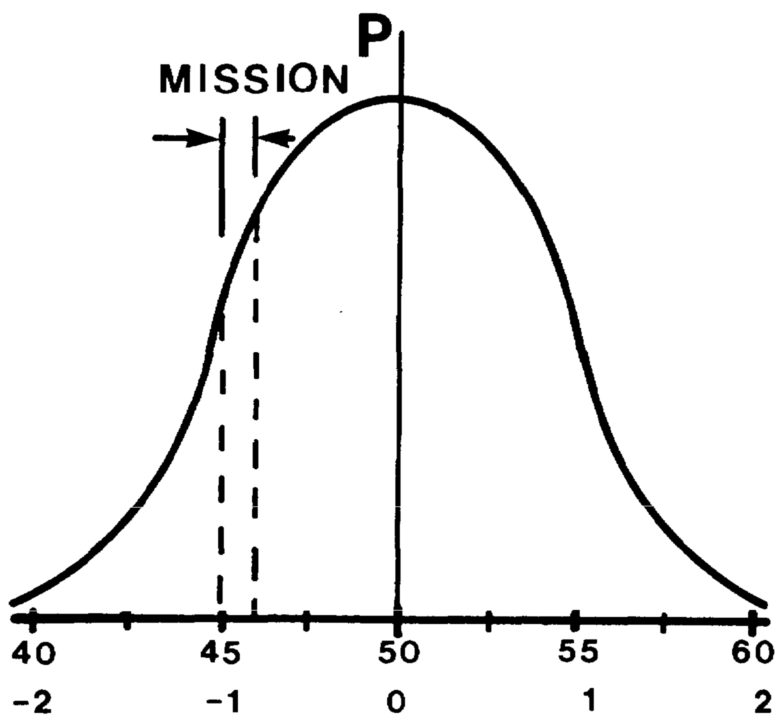
$$UIR = \frac{0.0300}{0.0397}$$

$$= 0.76$$

The parallel combination is less reliable than the single diode in this case because the second diode is more likely to fail the system by failing short than it is to save the system with diode 1 failed open.

## 121.00-9 Operation in the Wearout Region

1.



$$R(46000|45000) = R_u(46000|45000)\ R_w(46000|45000)$$

where $R_u(46000|45000) = e^{-1000\lambda_u}$

$$= 0.9990$$

and $$R_w(4600|4500) = \frac{R_w(46000)}{R_w(45000)}$$

$$= \frac{1 - F(\frac{46000 - 50000}{5000})}{1 - F(\frac{45000 - 50000}{5000})}$$

$$= \frac{F(0.8)}{F(1)}$$

$$= \frac{0.7881}{0.8413} \quad \text{(from Normal Distribution Table)}$$

$$= 0.9368$$

$\therefore$ Mission Reliability = 0.9990 x 0.9368

= 0.9359

2. See text.

3. - by replacing components preventatively before wearout
   - by installing redundant components
   - by increasing test frequency (passive systems only)

4. See text.

## 121.00-10 Some Modern Reliability Topics

### 1. Failure Mode and Effect Analysis

This is a suggested solution to the assignment question at the end of 121.00-10. It is not necessarily the only correct solution, and you may disagree with some of the conclusions. Your analysis should, however, have considered each of the failure modes detailed.

| Line Number | Item | Failure Mode | MTTF | Effect of Failure | Severity | Comments |
|---|---|---|---|---|---|---|
| 1 | Suction Filter | Blocked | $6 \times 10^2$ h | Greatly reduced oil supply. | 3 | Consider removal and replacement with tank strainer. |
| 2 | as 1 | Air leakage | $2 \times 10^5$ h | Reduced O/P. | 2 | as 1. |
| 3 | Pump #1 or #2 | Shut down | $5 \times 10^3$ h | Loss of backup. | 1 | Provide suction isolation valve for repair. |
| 4 | Electrical supply to pumps | Total loss | $2 \times 10^4$ h | Total loss of Lub oil Pressure. | 4 | Consider DC supplied back-up or alternative AC supplies. |
| 5 | Discharge NR valve | Open | $8 \times 10^8$ h | None in normal operation except back pressure on standby pump . | 2 | Only a problem if one pump out for maintenance, or if standby had to be isolated to prevent rotation. |
| 6 | as 5 | Shut | $6 \times 10^8$ h | Loss of standby pump availability. | 2 | |
| 7 | Discharge Filter | Heavy Leakage | $5 \times 10^5$ h | Reduced oil supply. Heavy oil loss. | 3 | Fit two in parallel with ganged change-over valves . |

| Line Number | Item | Failure Mode | MTTF | Effect of Failure | Severity | Comments |
|---|---|---|---|---|---|---|
| 8 | as 7 | Blocked | $2 \times 10^3$ h | Greatly reduced oil supply. | 3 | as 7 |
| 9 | Pump pressure gauge #1 or #2 | Loss of indication | $9 \times 10^3$ h | | 1 | Are 2 required? |
| 10 | as 9 | Burst | $8 \times 10^4$ h | Loss of pressure heavy loss of oil. | 4 | (a) move to pump side of NR valve<br>(b) fit isolation valve on root line<br>(c) use orifice on root line to reduce leak rate until isolated<br>OR use 1 gauge on suction of filter with isolation valve and orifice |
| 11 | Pressure gauge - filter discharge | Loss of indication | $9 \times 10^3$ h | No indication of pressure to bearings.<br>No indication of filter $\Delta P$ | 1<br>1 | Fit transducer and alarm<br>Fit 'pop up' $\Delta P$ alarm on filter |
| 12 | as 11 | Burst | $8 \times 10^4$ h | Loss of pressure heavy oil loss. | 4 | - use orifice to reduce leak rate<br>Consider replacement unit<br>(a) more reliable gauge, or<br>(b) transducer and remote gauge |

FMECA GRID

| failure rate \ severity | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 4 | | | 1 | |
| 3 | 3,9, 11 | | 8 | |
| 2 | | | | 4,10, 12 |
| 1 | | 5, 2, 6 | 7 | |

Design Changes

As a result of this analysis, the following design changes could be considered.

1. Remove the suction filter and replace by a coarse strainer in the tank.

2. Provide isolation valves on pump suctions.

3. Provide DC supplied back up pump.

4. Fit two discharge filters in parallel, with ganged change over valves and high differential pressure "pop-up" alarms.

5. Move the pumps discharge pressure gauges to the pump side of the NR valve. Fit isolation valves on the root line. Fit orifices into the root line to reduce leak rate until isolation, or fit one gauge at filter inlet with an isolation valve.

6. Fit pressure transducer and remote gauge, or alarm, at filter outlet.

L. Haacke
R. Malcolm